Example solution: decomposing a solution into set of relations which are in BCNF

This is an example solution which shows what is demanded to get full points from an exercise or exam problem which asks to justify why a given relation is not in BCNF and to decompose it into set of relations which are in BCNF. Please note that only one of the possible solutions is presented below (instead of using the second closure, it is possible to do the first decomposition by using the first or the third closure).

Problem
Consider a relation R with schema $R(A, B, C, D, E)$ and functional dependencies $A \rightarrow B$, $B \rightarrow D\ E$, and $C \rightarrow E$. Explain why this relation is not in Boyce-Codd normal form (BCNF). Decompose the relation using the BCNF decomposition algorithm taught in this course and in the text book. Give a short justification for each new relation. Continue the decomposition until the final relations are in BCNF. Explain why the final relations are in BCNF.

Solution

- Calculate the closures of the left sides of the dependencies:

$$\begin{aligned} \{A\}^+ &= \{A, B, D, E\} \\ \{B\}^+ &= \{B, D, E\} \\ \{C\}^+ &= \{C, E\} \end{aligned}$$

- If R were in BCNF, all given closures should contain all attributes of R. Because none of them does, none of the left sides of the given dependencies is superkey of R and R is not in BCNF.
- Decompose R by using the second dependency: $R_1(B, D, E)$ (closure of B) and $R_2(A, B, C)$ (B and those attributes of R which do not belong to the closure of B).
- Check whether the new relations are in BCNF
- R_1 has only a non-trivial dependency $B \rightarrow D\ E$. Calculate the closure of the left side:

$$\{B\}^+ = \{B, D, E\}$$

  The closure contains all attributes of R_1. Thus, B is superkey of R_1 and R_1 is in BCNF.
- R_2 has a non-trivial dependency $A \rightarrow B$. Calculate the closure of the left side:

$$\{A\}^+ = \{A, B\}$$

  The closure does not contain C. Thus, A is not superkey of R_2 and R_2 is not in BCNF. We must decompose R_2 again.
- Decompose $R_2(A, B, C)$: We obtain $R_3(A, B)$ (closure of A) and $R_4(A, C)$ (A and those attributes of R_2 which are not in closure of A).
- Check whether R_3 and R_4 are in BCNF.
- R_3 has only a non-trivial dependency $A \rightarrow B$. Calculate the closure of the left side:

$$\{A\}^+ = \{A, B\}$$

  Because the closure contains all attributes of R_3, A is the superkey and R_3 is in BCNF.
- R_4 has no non-trivial functional dependencies. Thus R_4 is in BCNF.
- The final relations are $R_1(B, D, E)$, $R_3(A, B)$, and $R_4(A, C)$.